Invertible Transformation

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(1) **One-one Transformation:-** let $T: V \to W$ be a linear Transformation. Then T is called one-one if for all $x \neq y \Rightarrow T(x) \neq T(y)$ or

$$x = y \Rightarrow T(x) = T(y).$$

another name is injective transformation;

(2) **onto (surjective) Transformation:** let $T: V \to W$ be a linear transformation. Then T is called onto iff for each $w \in W$, $\exists\ v \in V$ s.t $T(v) = w$ or W = Range of T.

(3) **Invertible operator:** A linear transformation $T: V(F) \to V(F)$ is said to be invertible operator iff $\exists$ an operator $S: V(F) \to V(F)$ such that $TS = I = ST$, where I is the identity operator.

Here S is the inverse of T

and it is denoted as $\boxed{T^{-1} = S}$

Singular Transformation:- A linear transformation $T: V \to W$ is said to be singular iff the null space of T contains at least one non-zero vector.

Thus if $v \neq 0 \Rightarrow T(v) = 0$ for some $v \in V$

Then T is called singular transformation.

Non-singular Transformation: A linear transformation $T: V \to W$ is said to be non-singular iff the null space of T is zero space $\{0\}$, i.e. the null space consists only the zero element, i.e If $T(v) = 0 \Rightarrow v = 0$ for all $v \in V$

or if $v \neq 0 \Rightarrow T(v) \neq 0$ for all $v \in V$

Then T is said to be non-singular.

To find T^{-1}

1. $TS = ST = I$ Then we $\boxed{T^{-1} = S}$

1-1 onto (i.e Bijective)

1. $TS = ST = I$

2. T is one-one and onto (i.e Bijective)

then find T^{-1}

3. T is invertible iff T is non-singular

Q: let T be a linear operator on R^3 defined by $T(x, y, z) = (x-2y-z, y-z, x)$, Show that T is invertible, and find T^{-1}.

Sol: we know that T is investible iff T is non-singular.

To show T is non-singular.

let $T(x, y, z) = (0,0,0)$ for some $v = (x,y,z) \in R^3$

$$\Rightarrow (x-2y-z, y-z, x) = (0,0,0)$$

$$\Rightarrow \begin{aligned} x-2y-z &= 0 \\ y-z &= 0 \\ x &= 0 \end{aligned}$$

To solve $x=0, y=0, z=0$.

i.e $(x, y, z) = (0,0,0)$

i.e $T(x,y,z) = (0,0,0) \Rightarrow (x,y,z) = (0,0,0)$

T is non singular.

$\Rightarrow$ T is invertible operator on R^3

Now To find T^{-1}

let $T(x,y,z) = (a,b,c)$

$\Rightarrow (x-2y-z, \; y-z, \; x) = (a, b, c)$

$\therefore$ $x-2y-z = a$ ——— (1)

$y-z = b$ ——— (2)

$x = c$ ——— (3)

$\therefore$ $x = c$, from (1), $-2y-\cancel{z} = a-c$

from (2) $\underline{-y+\cancel{z} = -b}$

$-3y = a-b-c$

$$y = \frac{a-b-c}{-3} = \frac{-a+b+c}{3}$$

$\therefore$ using the value of y, we get

$z = y-b$

$$= \frac{-a+b+c}{3} - b$$

$$= \frac{-a+b+c-3b}{3}$$

$$= \frac{-a-2b+c}{3}$$

$\therefore$ T^{-1} is given by $T^{-1}(a,b,c) = (x,y,z)$

$$T^{-1}(a,b,c) = \left(c, \frac{-a+b+c}{3}, \frac{-a-2b+c}{3}\right)$$

Ans

H.W.

(1). Let T be a linear operator on R^3 defined by $T(x,y,z) = (2x, 4x-y, 2x+3y-z)$

Then show that T is invertible and find T^{-1}!

(2) Let $T: V_3(R) \to V_3(R)$ be defined by

$$T(x,y,z) = (3x, x-y, 2x+y+z),$$

Then prove that T is invertible. and find T^{-1}!

Sol: We know that T is invertible iff T is non-singular

Sol: We Know that T is invertible iff T ...

Now To show that T is non-singular,

let $T(v) = 0$ for $v = (x, y, z) \in R^3$

i.e $T(x, y, z) = 0$

$(3x, x-y, 2x+y+z) = (0, 0, 0)$

$3x = 0 \Rightarrow \boxed{x = 0}$

$x - y = 0 \Rightarrow y = x = 0 \Rightarrow \boxed{y = 0}$

$2x + y + z = 0 \Rightarrow z = 0 - 2x - y$

$= 0 - 0 - 0$

$\boxed{z = 0}$

$\therefore v = (x, y, z) = (0, 0, 0)$

$\therefore T(v) = T(x, y, z) = (0, 0, 0) \Longrightarrow (x, y, z) = (0, 0, 0)$

$\therefore T$ is non-singular.

$\Rightarrow T$ is invertible operator on R^3.

Now To find T^{-1},

... $z) = (a, b, c)$

Let $T(x, y, z) = (a, b, c)$

$$\Rightarrow (3x,\ x-y,\ 2x+y+z) = (a, b, c)$$

$$\Rightarrow \quad 3x = a \quad \Rightarrow \quad x = a/3$$

$$x - y = b \quad \Rightarrow \quad y = x - b = \frac{a}{3} - b$$

$$= \frac{a-3b}{3}$$

$$2x+y+z = c$$

$$\rightarrow z = c - 2x - y$$

$$= c - 2\frac{a}{3} - \left(\frac{a-3b}{3}\right)$$

$$= \frac{3c - 2a - a + 3b}{3}$$

$$= \frac{-3a + 3b + 3c}{3}$$

$$= (-a + b + c)$$

$$\therefore \quad T(x, y, z) = (a, b, c)$$

i.e. $T^{-1}(a, b, c) = (x, y, z)$

$$= \left(\frac{a}{3},\ \frac{a}{3} - b,\ c - a + b\right)$$

Which is the required inverse of T.

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